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Strength of Initially Wavy Lattice Columns

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Axial compressive strengths are determined for lattice columns that are initially wavy in both local and overall modes. When both waviness modes are present, the strength is limited by the onset of divergent amplification of the waviness, with associated reductions in structural stiffness. For such divergent but non-bifurcated states of equilibrium, the compressive strength is less than either the classical local or overall buckling strengths. These strength reductions are anticipated to be significant for lightweight space columns of large slenderness ratio, such as the diagonal spars of an 800-m square solar sailing spacecraft which prompted this investigation.

Nomenclature

A_ℓ = longeron cross-sectional area, same for each longeron
C_i = coefficient used in equations for F_i; see equation in text for C_i
E = Young's modulus for longeron material
E_i = effective Young's moduli for an ith wavy longeron; $i = 1, 2$, and 3; see Fig. 2 for numbering convention
F_i = axial force in an ith longeron
F_i^γ = portion of midlength force in ith longeron necessary to equilibrate bending moment
F_{CR} = Euler buckling strength of a longeron
f_i = normalized axial force in ith longeron; $f_i = F_i / F_{CR}$
I_B = cross-sectional moment of inertia for lattice column
K = ratio of lattice column Euler strength to three times the longeron Euler strength
L = overall length of lattice column
ℓ = baylength of column
M_γ = internal bending moment at column midlength
P = externally applied axial load on column
P_B = Euler buckling strength of lattice column with bending stiffness EI_B
P_{EU} = Euler buckling strength of lattice column with ideally straight longerons
p_B^* = P/P_B; applied load as normalized to P_B
p_{EU}^* = P/P_{EU}; applied load as normalized to P_{EU}
$p_{\max}^*$ = maximum value of p_{EU}^*
$\bar{p}_B$ = P_B / P_{EU}; ratio of actual to ideal Euler strengths
R = radius of circle through ideal centerlines of the three longerons of the lattice column
x = coordinate along ideal axis of either the lattice column or any one of its longerons
y = initial deflection shape of either lattice column or any of its longerons
y_L = amplitude of initial waviness of axis of lattice column
y_ℓ = amplitude of initial waviness of longeron
y_F = deflected shape of wavy longeron under axial load F
y_P = deflected amplitude of wavy column under axial load P
γ = angle between a column face and a moment vector for bending column to initial overall waviness; see Fig. 2
δ = total shortening of a longeron
δ_y = longeron shortening due to bending
ρ_ℓ = longeron radius of gyration

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Introduction

THIS study was prompted by a need to predict the axial compressive strength of lattice columns which were being considered for the diagonal spars of an 800-m square solar sailing spacecraft. Because they were very lightly loaded, these spars had large slenderness ratios (length-to-radius of gyration) which indicated they would be vulnerable to initial waviness. Therefore, a method was needed to predict the axial compressive strength of such spars when they are wavy, both overall and locally in their longerons.

Crawford and Hedgepeth[1] analyzed effects of local longeron waviness on the strength of lattice columns, and Mikulas[2] analyzed effects of overall waviness on similar columns. Thus, both overall and local waviness, treated separately, have been found to reduce lattice column strength significantly. Van der Neut[3] analyzed the effects of both

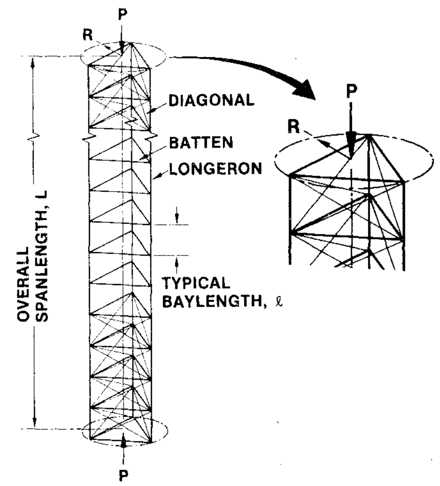


Fig. 1 Lattice column, axially loaded.

types of waviness being present simultaneously for columns built-up of plates which develop post-buckling stiffness. He found only "slight" reduction in column strength due to the overall waviness. However, because the longerons of lattice columns have virtually no post-buckling stiffness, it was inferred that Van der Neut's conclusion does not apply to the present problem. Indeed, Ref. 1 showed that effects of initial local waviness are much different for lattice columns than for ones built-up of plates. Thus, it was concluded that both types of waviness could have significant effects and must, therefore, be analyzed for the case where they are present simultaneously.

The present methods of analysis are approximate, as noted later. Also, the analysis is idealized inasmuch as the longeron initial waviness, for instance, is assumed to be sinusoidal and of constant wavelength and amplitude. It was judged that in the face of such idealization, more involved and exact strength analysis than presented here is not warranted.

Analysis

The principal assumptions made in this analysis are as follows:

1) The column analyzed for this application is the equilateral triangular structure shown in Fig. 1, simply supported with spanlength L, equal baylengths ℓ, and nominal radius R. The longerons are identical and simply supported at spacings ℓ by the battens and diagonals of the structure.

2) Axial compressive force P is the only external loading on the column.

3) The initial waviness of the axis of the column assembly is $y=y_L\sin(\pi x/L)$. Waviness direction is defined by an angle γ between a column face and a moment vector for producing the waviness by bending.

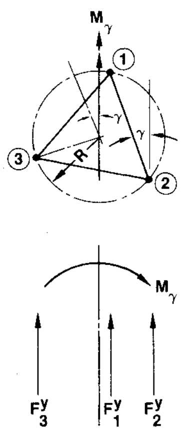
4) Each longeron axis is initially wavy as defined by $y=y_\ell\sin(\pi x/\ell)$.

5) In reaction to P the column axis deflects in the same mode and direction as its initial waviness. Note (Ref. 4) that this is the exact reaction for small deflections of a sinusoidally wavy column of uniform bending stiffness along its length and simply-supported. However, it is only an approximation for the present case where the bending stiffness is nonuniform (see later development).

6) The column has a uniform bending stiffness along its length which is equal to the stiffness calculated for its midlength.

7) All stresses in the column are elastic.

8) Effects of finite transverse shearing stiffness are negligible.



① = LONGERON NUMBERS

F_i^y = LONGERON FORCES, POSITIVE IN COMPRESSION

Fig. 2 Bending moment on column and longeron reactions.

Before beginning the detailed analysis, it is noted that a key parameter affecting the stiffness of the structure is the ratio of waviness amplitude to cross-sectional radius of gyration; i.e., $\sqrt{2}y_L/R$ for the overall column and y_ℓ/ρ_ℓ for the longerons. Fabrication and inspection technology appear to be such that the ratio of waviness amplitude to length is somewhat of a constant that is independent of the slenderness ratio. But, because (for instance, for a longeron) the key parameter is given by

$$\frac{y_\ell}{\rho_\ell}=\frac{y_\ell}{\ell}\frac{\ell}{\rho_\ell} \tag{1}$$

where ρ_ℓ = longeron radius of gyration, then large slenderness ratios (ℓ/ρ_ℓ) will exacerbate the initial waviness problem. Also note that efficiently designed lattice columns will tend to have approximately equal overall and local slenderness ratios.

Consistent with the above-listed assumptions, the buckling strength for the overall column is

$$P_B=\pi^2 EI_B/L^2 \tag{2}$$

where EI_B is the assumed uniform bending stiffness of the column. Because this initially wavy column will bend under axial force P, the longerons will be unequally compressed. As will be shown later, the effective axial stiffness of an initially wavy longeron depends upon its axial compression. Therefore, at the midspan of the column, the longerons will have unequal axial stiffness which are defined later.

By a straightforward application of statics, with the usual assumption that plane sections of the column remain plane during bending, the following formula is derived for the bending stiffness EI_B of a column with longerons of unequal effective Young's moduli E_1, E_2 and E_3.

$$EI_B=\frac{0.75EA_\ell R^2}{\frac{E}{E_1}\frac{E}{E_2}+\frac{E}{E_1}\frac{E}{E_3}+\frac{E}{E_2}\frac{E}{E_3}}\left[\frac{E}{E_1}(\sin\gamma+\sqrt{3}\cos\gamma)^2\right.$$

$$\left.+\frac{E}{E_2}(\sin\gamma-\sqrt{3}\cos\gamma)^2+4\frac{E}{E_3}\sin^2\gamma\right] \tag{3}$$

The total axial force F_i acting on an ith longeron is

$$F_i=(P/3)+F_i^y \tag{4}$$

where F_i^y are the midlength longeron forces that equilibrate the bending moment produced by P and the overall waviness. From Timoshenko,[4] the total deflection y_P produced by P in the direction of y_L is

$$y_P=\frac{y_L}{1-p_B^*} \tag{5}$$

where $p_B^*=P/P_B$. In all subsequent equations p_B^* will be expressed in terms of P_{EU}, the ideal Euler strength of the column. That is,

$$p_B^*=\frac{P}{P_{EU}}\Big/\frac{P_B}{P_{EU}}=p_{EU}^*/\bar{p}_B \tag{6}$$

Accordingly, the internal bending moment M_γ (see Fig. 2) is

$$M_\gamma=Py_L/(1-p_{EU}^*/\bar{p}_B) \tag{7}$$

Longeron loads F_i^y which equilibrate M_γ are determined from statics as follows. For axial equilibrium,

$$\Sigma F_i^y=0 \tag{8}$$

For equilibrium of moments in the direction of M_γ (see Fig. 2),

$$F_1^\gamma R\sqrt{3}\sin\gamma + F_3^\gamma R\frac{\sqrt{3}}{2}(\sin\gamma+\sqrt{3}\cos\gamma)+M_\gamma=0 \quad (9)$$

And, for equilibrium of moments in the direction perpendicular to M_γ,

$$F_1^\gamma R\cos\gamma+\frac{F_3^\gamma}{2}(\cos\gamma-\sqrt{3}\sin\gamma)=0 \quad (10)$$

F_i is determined as follows from the preceding equations:

$$F_i=(P/3)C_i \quad (11)$$

where C_i are

$$C_1=1+\frac{(y_L/R)(\cos\gamma-\sqrt{3}\sin\gamma)}{1-p_{EU}^*/\bar{p}_B}$$

$$C_2=1+\frac{(y_L/R)(\cos\gamma+\sqrt{3}\sin\gamma)}{1-p_{EU}^*/\bar{p}_B}$$

$$C_3=1-\frac{(y_L/2R)\cos\gamma}{1-p_{EU}^*/\bar{p}_B}$$

The axial shortening of an initially wavy longeron due to bending is

$$\delta_y=\frac{1}{2}\int_0^\ell (y_F')^2\mathrm{d}x-\frac{1}{2}\int_0^\ell (y')^2\mathrm{d}x \quad (12)$$

where

$$y_F=\frac{y_\ell}{1-f_i}\sin\frac{\pi x}{\ell},\quad y=y_\ell\sin\frac{\pi x}{\ell},\quad f_i=\frac{F_i}{F_{CR}}$$

and

$$F_{CR}=\pi^2EA_\ell\rho_\ell^2/\ell^2$$

By performing the indicated integrations, δ_y is determined as

$$\delta_y=\frac{\pi^2y_\ell^2}{4\ell}\left[\frac{1}{(1-f_i)^2}-1\right] \quad (13)$$

The total shortening δ inclues that owing to longeron axial strain. Accordingly,

$$\delta=\delta_y+(F_i\ell/A_\ell E) \quad (14)$$

The effective Young's modulus is then

$$E_i=\frac{\ell}{A_\ell}\frac{\mathrm{d}F_i}{\mathrm{d}\delta} \quad (15)$$

By taking the indicated derivative of the above equations, the effective moduli are determined as

$$\frac{E}{E_i}=1+\frac{(y_\ell/\rho_\ell)^2}{2(1-f_i)^3} \quad (16)$$

Note that

$$f_i=(P/3F_{CR})C_i=p_{EU}^*C_iK \quad (17)$$

where

$$p_{EU}^*=P/P_{EU}$$

And, since

$$P_{EU}=3\pi^2EA_\ell R^2/2L^2 \quad (18)$$

then

$$K=(P_{EU}/3F_{CR})=\tfrac{1}{2}(\ell/L)^2(R/\rho_\ell)^2 \quad (19)$$

The value of K is 1.0 for an efficient design of an ideally straight lattice column where the longerons buckle at the same load as for overall Euler buckling.

Thus, the previously defined parameter $\bar{p}_B$, ratio of actual-to-ideal column bending stiffnesses, is

$$\bar{p}_B=\frac{EI_B}{1.5EA_\ell R^2}=\frac{1}{2}\left(\frac{E}{E_1}\frac{E}{E_2}+\frac{E}{E_1}\frac{E}{E_3}+\frac{E}{E_2}\frac{E}{E_3}\right)^{-1}$$
$$\times\left[\frac{E}{E_1}(\sin\gamma+\sqrt{3}\cos\gamma)^2+\frac{E}{E_2}(\sin\gamma-\sqrt{3}\cos\gamma)^2+4\frac{E}{E_3}\sin^2\gamma\right] \quad (20)$$

where the E/E_i ratios are expressed by the previous equations in terms of the bending stiffness parameter $\bar{p}_B$, the applied load parameter p_{EU}^*, the design parameter K, the waviness, parameters y_ℓ/ρ_ℓ and y_ℓ/R, and the direction angle γ.

It was calculated in the present investigation that y_L has its most deleterious effect in reducing EI_B when $\gamma=\pi/3$. At this value of γ, the bending moment M_γ is reacted by compression in one longeron and equal tensions in the other two; each tension is half the compression in magnitude. For this case $\gamma=\pi/3$, the foregoing equations are combined into the following relatively simple relationship among the remaining variables:

$$\frac{1.5}{\bar{p}_B}=\frac{1}{2}\left\{1+\frac{(y_\ell/\rho_\ell)^2}{2\left[1-p_{EU}^*K\left(1-\frac{y_L/R}{1-p_{EU}^*/\bar{p}_B}\right)\right]^3}\right\}+1$$
$$+\frac{(y_\ell/\rho_\ell)^2}{2\left[1-p_{EU}^*K\left(1+\frac{2y_L/R}{1-p_{EU}^*/\bar{p}_B}\right)\right]^3} \quad (21)$$

Solutions to this equation are shown in Fig. 3 for $K=1$, as for optimum design of a non-wavy column, and for $y_\ell/\rho_\ell=0.30$. The solutions show the following:

1) As p_{EU}^* (the applied load parameter) increases from zero, $\bar{p}_B$ (the bending stiffness parameter) decreases from its maximum value. That is, EI_B decreases as local and overall bending is increased in reaction to P.

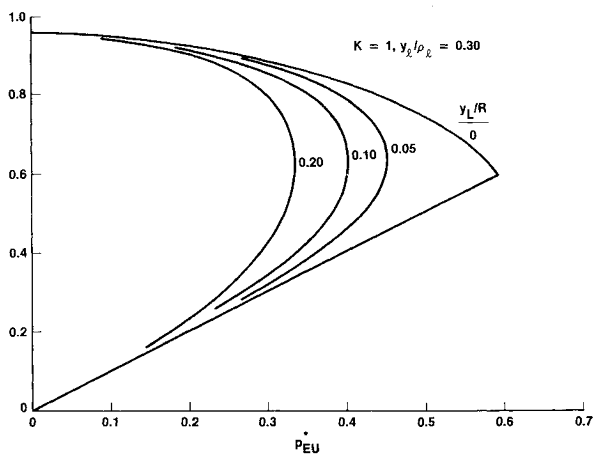


Fig. 3 Bending stiffness ratio $\bar{p}_B$ vs p_{EU}^* and waviness parameters.

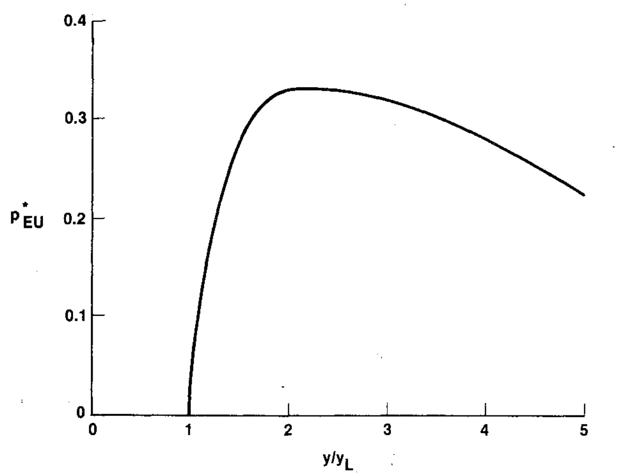


Fig. 4 Lateral deflection vs applied load for $y_\ell/\rho_\ell = 0.30$ and $y_L/R = 0.20$.

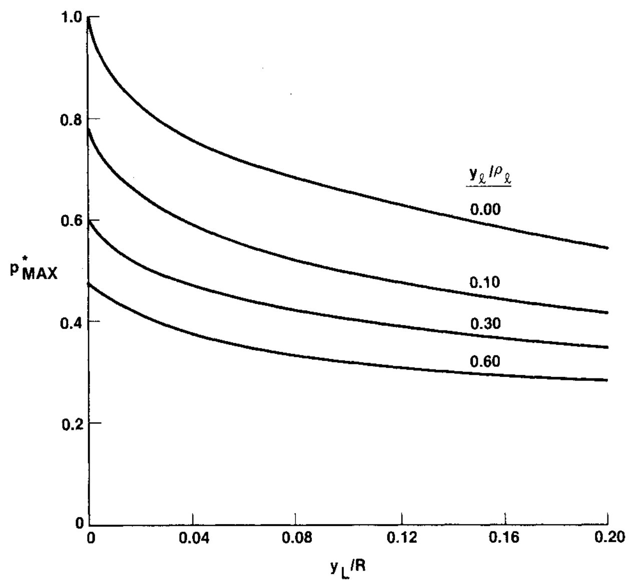


Fig. 5 Maximum compressive strength of column vs waviness parameters y_ℓ/ρ_ℓ and y_L/R; $K=1$.

2) When $y_L = 0$, the maximum value of p^*_{EU} is $\bar{p}_B$, the Euler buckling strength as reduced by the longeron local waviness. This case of $K=1$ and $y_L = 0$ corresponds to the analysis of lattice columns presented in Ref. 1.

3) When $y_L > 0$, the maximum value of p^*_{EU} ($p^*_{\max}$) is always less than $\bar{p}_B$. In this case $p^*_{\max}$ is not the usual buckling phenomenon, a bifurcation of equilibrium configurations. Instead, $p^*_{\max}$ is simply the maximum load that can be withstood before subsequent bending and decrease of EI_B proceeds divergently.

The physical significance of the last observation is, perhaps, made clearer by considering that the midspan deflection y of an initially wavy column is[4] (for a column of uniform bending stiffness),

$$y = y_L / 1 - p^*_{EU}/\bar{p}_B \tag{22}$$

Accordingly, y is positive and bounded only when $p^*_{EU} < \bar{p}_B$. Figure 4 shows y/y_L vs p^*_{EU} for the solutions given in Fig. 3 for $y_L/R = 0.20$. It is clear in Fig. 4 that the load that can be applied is limited by the onset of the divergent deflection condition.

Figure 5 shows $p^*_{\max}$ vs y_ℓ/ρ_ℓ and y_L/R for the example case of $K=1$. It is seen that only small amounts of either type of initial waviness cause significant reduction in $p^*_{\max}$. However, when these wavinesses are large, then $p^*_{\max}$ is not so sensitive to further changes in those wavinesses.

Example Design

The spars of the previously mentioned solar sailing spacecraft were so designed that $\ell/\rho_\ell = 214$ and $L/R = 197$. Estimates of the contributions of various sources to the initial wavinesses of those spars gave $y_\ell/\ell = 1/2500$ and $y_L/L = 1/1000$. Accordingly, $y_\ell/\rho_\ell = 0.086$ and $y_L/R = 0.197$. Figure 5 shows that for these wavinesses $p^*_{\max} \simeq 0.43$.

Concluding Remarks

Combined effects of overall and longeron initial waviness on the axial strength of lattice columns have been analyzed and found to be significant. The presence of either type of waviness exacerbates the deleterious effects of the other. When both types of waviness are present, the axial strength is limited by the onset of divergent column deflection and accompanying loss of bending stiffness rather than by a classical stability failure.

In view of the large number of idealistic assumptions it would be interesting to compare these results with more exact analysis or experiment results. However, neither appear to exist at the present time.

The strength of a diagonal spar design for a solar sailing spacecraft was found to be quite vulnerable to these initial waviness effects. They are likely to be present also in future large space structures and should be accounted for.

For the solar sailing application, effects of uniform lateral loading were included without changing the character of the problem. However, that slight complication was deleted from this presentation for clarity.

References

[1] Crawford, R. F. and Hedgepeth, J. M., "Effects of Initial Waviness on the Strength and Design of Built-Up Structures," *AIAA Journal*, Vol. 13, May 1975, pp. 672-675.

[2] Mikulas, M. M., "Structural Efficiency of Long, Lightly-Loaded Truss and Isogrid Columns for Space Applications," NASA TM 78687, July 1978.

[3] Van der Neut, A., "The Interaction of Load Buckling and Column Failure of Thin-Walled Compression Members," *Proceedings of the Twelfth International Congress of Applied Mechanics*, edited by M. Hetenyi and W. G. Vincente, Springer, Berlin, 1969, pp. 389-399.

[4] Timoshenko, S., *Theory of Elastic Stability*, McGraw-Hill, New York, 1936, pp. 31-33.